SECTION-1

Section-I (Single Correct Choice Type) contains 8 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), out of which ONLY ONE is correct. For each question in Section I, you will be awarded 3 marks if you have darkened only the bubble corresponding to the correct answer and zero mark if no bubbles are darkened. In all other cases, minus one (-1) mark will be awarded.

1) The number of solutions of the equation $8\left[x^2-x\right]+4\left[x\right]=13+12\left[\sin x\right]$ is (where [.] represents greatest integer function)

A) 0 B) 2 C) 4 D) 6

2) If $0<\alpha<\frac{\pi}{2}$ and $f(\alpha)=\pi^{\sec^2\alpha}.\cos^2\alpha+\pi^{\operatorname{cosec}^2\alpha}.\sin^2\alpha$, then

A) $f(\alpha)<\pi^2$ B) $f(\alpha)>\pi^2$ C) $f(\alpha)<\pi$ D) $f(\alpha)=\pi^2$

3) The sequence $a_1, a_2, a_3, ...$ satisfies $a_1=19$, $a_{99}=99$ & $\forall n\geq 3, a_n$ is the arithmetic mean of the first $(n-1)$ terms, then $a_2=$

A) 179 B) 81 C) 79 D) 181

4) If the graphs of the functions $y=ln\,x$ & $y=ax$ intersect at exactly two points, then

A) $a\in(0,e)$ B) $a\in\left(0,\frac{1}{e}\right)$ C) $a\in(-e,1)$ D) $a\in(1,e)$

5) $\int_0^{\pi/2}\frac{dx}{\left(4\cos^2x+9\sin^2x\right)^2}=$

A) $\frac{11\pi}{864}$ B) $\frac{13\pi}{864}$ C) $\frac{17\pi}{864}$ D) $\frac{97\pi}{864}$

6) In a $\Delta^{le}ABC$, if $A=(1,2)$ and internal angle bisectors through B and C are $y=x$ and $y=-2x$. The in-radius 'r' of $\Delta^{le}ABC$ is equal to

A) $\frac{1}{\sqrt{3}}$ B) $\frac{2}{3}$ C) $\frac{1}{3}$ D) $\frac{1}{\sqrt{2}}$

$\left(x\cot y+ln(\cos x)\right)dy+\left(ln(sin\,y)-y\tan x\right)dx=0$ is

A) $(\sin x)^{y}(\cos y)^{x}=c$ B) $(\sin y)^{x}(\cos x)^{y}=c$

C) $(\sin x)^{y}(\sin y)^{x}=c$ D) $(\cot x)^{y}(\cot y)^{x}=c$

8) There are $'n'$ coplanar straight lines, no two being parallel and no three are concurrent. The number of different new straight lines that will be formed by joining the intersection points of the given lines are

A) $\frac{n(n-1)(n-2)(n-3)}{8}$ B) $\frac{(n-1)(n-2)}{2}$

C) $\frac{n(n-1)}{4}$ D) $\frac{n(n+1)(n+2)}{4}$

SECTION-2

Section-II (Multiple Correct Choice Type) contains 5 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), out of which ONE OR MORE may be correct. For each question in Section II, you will be awarded 3 marks if you darken only the bubble corresponding to the correct answer and zero mark if no bubbles are darkened. Partial marks will be awarded for partially correct answers. No negative marks will be awarded in this Section.

9) If $a,b,c,d\in R$ satisfies $a^2+b^2+c^2+d^2-ab-bc-cd-d+\frac{2}{5}=0$, then

A) $c=a+b$ B) $b=\frac{d}{2}$ C) $a=\frac{c}{3}$ D) $d=a+c$

10) The set $'S'$ of all real $'x'$ for which $\left(x^2-x+1\right)^{x-1}<1$ contains

A) $(-5,-1)$ B) $(-1,1)$ C) $(-1,0)$ D) $(-3,1)$

11) Let $g(x)=f(\tan x)+f(\cot x)\,\forall x\in\left(\frac{\pi}{2},\pi\right)$. If $f''(x)<0\ \forall x\in\left(\frac{\pi}{2},\pi\right)$, then

A) $g(x)$ is increasing in $\left(\frac{\pi}{2},\frac{3\pi}{4}\right)$ B) $g(x)$ is increasing in $\left(\frac{3\pi}{4},\pi\right)$

C) $g(x)$ is decreasing in $\left(\frac{3\pi}{4},\pi\right)$ D) $g(x)$ has local maximum at $x=\frac{3\pi}{4}$

12) The equation of a straight line which is tangent to one point and normal to the other point

A) $\sqrt{2}x-y=\frac{31\sqrt{2}}{27}$

B) $\sqrt{2}x-y=\frac{39\sqrt{2}}{27}$

C) $\sqrt{2}x+y=\frac{31\sqrt{2}}{27}$

D) $\sqrt{2}x+y=\frac{39\sqrt{2}}{27}$

13) Let $f:R\to R$ be a function defined by $f(x+1)=\frac{f(x)-5}{f(x)-3}\ \forall x\in R$.

Then which of the following statement(s) is/are true

A) $f(x+2)=f(x)$

B) $f(x+4)=f(x)$

C) $f(x+6)=f(x)$

D) $f(x+8)=f(x)$

SECTION-3

Section-III (Paragraph Type) contains 2 paragraphs. Based upon the first paragraph 3 multiple choice questions and based upon the second paragraph 2 multiple choice questions have to be answered. Each of these questions has 4 choices (A), (B), (C) and (D) out of which ONLY ONE is correct. For each question in Section III, you will be awarded 3 marks if you darken only the bubble corresponding to the correct answer and zero mark if no bubbles are darkened. In all other cases, minus one (-1) mark will be awarded.

P) Consider a ΔABC with $A, B \& C$ represented by the complex numbers $z_1, z_2 \& z_3$ respectively. The in-circle of ΔABC with centre O (the origin) touches the sides $BC, CA \& AB$ at the points $z_4, z_5 \& z_6$ respectively. AO cuts the circum circle of ΔABC at z.

14) The point z_3 is

A) $\frac{1}{z_4}+\frac{1}{z_5}$

B) $\frac{1}{2}(z_4+z_5)$

C) $\frac{2}{\left(\frac{1}{z_4}+\frac{1}{z_5}\right)}$

D) $\frac{2}{z_4+z_5}$

15) $\frac{|z-z_2|}{|z|}+\frac{|z|}{|z-z_3|}$ equals

A) 1 B) 2 C) $\frac{1}{2}$ D) 4

16) $\left(\frac{z-z_2}{z}\right)\left(\frac{z_3-z_5}{z_3-z_4}\right)$ equals

A) 1 B) -1 C) 2 D) -2

P) Let ABC is a acute angled triangle and AD is perpendicular from A on side BC.

AD is produced and it is intersecting at point P with circum circle of ΔABC. H is the orthocenter of ΔABC and R is the circum radius of ΔABC

17) Area of $\Delta BHP =$

A) $2R^2 \cos^2 B \sin^2 C$

B) $2R^2 \cos B \sin B \sin 2C$

C) $2R^2 \sin^2 B \cos^2 C$

D) $2R^2 \cos^2 B \sin 2C$

18) Radius of circum circle of ΔAHC is

A) $\frac{R}{2}$ B) R C) $2R$ D) $3R$

SECTION-4

Section-IV (Integer Type) contains 10 questions. The answer to each question is a single digit integer ranging from 0 to 9. The correct digit below the question number in the ORS is to be bubbled. For each question in Section IV, you will be awarded 3 marks if you darken the bubble corresponding to the correct answer and zero mark if no bubble is darkened. No negative marks will be awarded for in this Section.

19) The sum of the radii of the smallest and largest circles passing through $\left(a, \frac{1}{a}\right)$ and touching the circle $4x^2+4y^2-4x-8y-31=0$ where $'a'$ is the least ratio of the sides of the regular $'n'$ sided polygon inscribing and circumscribing the unit circle is

20) Let $n_1, n_2, n_3, ...$ be increasing sequence of natural numbers each of which leaves remainder $'r'$ when it is divided by $(r+1)$ for $r = 2, 3, 4, ..., 9$. When n_{2008} is divided by 11, the remainder is

21) The number of planes that are equidistant from four non coplanar points is are all different digits & $\alpha\beta, \delta\beta$ are two digit numbers & $\gamma\gamma\gamma$ is a three digit

number, and the trace of the matrix $A=\begin{bmatrix}\alpha & 1 & 2 & 0\\ 0 & \beta & 1 & 1\\ 0 & 0 & \gamma & 3\\ 1 & 1 & 0 & \gamma\end{bmatrix}$ is a, then $\frac{a}{7}$ is equal to

23) If $'x'$ is positive and $x-[x],[x]$ and $'x'$ are in G.P., (where [.] denotes greatest integer function), then $\left(2x-2[x]+1\right)^2$ is equal to

24) For a triangle $ABC, R=\frac{5}{2}$ & $r=1$. Let $'I'$ be the in-centre of the triangle and D, E, F be the feet of the perpendiculars from $'I'$ to BC, CA, AB respectively. Then the value of $\frac{IA.IB.IC}{2ID.IE.IF}$ is equal to

25) If the area bounded by the curves $|y|=e^{-|x|}-\frac{1}{2}$ and $\frac{|x|+|y|}{2}+\left|\frac{|x|-|y|}{2}\right|\le 2$ is $'k'$ units, then $\left[\frac{k}{14}\right]$ is equal to (where [.] denotes greatest integer function)

26) If the coefficient of $a^8b^4c^9d^9$ in $\left(abc+bcd+cda+dab\right)^{10}$ is $\frac{K!}{2}$, then $K=$

27) If the maximum value of $'p'$ such that 3^p divides $99\times 97\times 95\times ......\times 51$ is $2K$,

then $K=$

28) If two distinct chords of a parabola $y^2=4ax$ passing through $(a, 2a)$ are bisected by the line $x+y=1$, and $4a$ is a natural number, then the maximum length of the

latus–rectum is

Key and Solutions

1	A	**2**	B	**3**	A	**4**	B	**5**	B	**6**	D
7	B	**8**	A	**9**	A,B,C,D	**10**	A,C	**11**	A,C,D	**12**	A,C
13	B,D	**14**	C	**15**	B	**16**	A	**17**	D	**18**	B

25 1 **26** 7 **27** 7 **28** 3

1) R.H.S. in an odd number always, L.H.S. is even number always.

2) $A.M > G.M$

$f(\alpha) \geq \pi^{2/\sin 2\alpha}.\sin 2\alpha$

$f(x) = x\pi^{2/x}$ is a decreasing function for $x \in (0,1) \Rightarrow f(x) > \pi^2$

3) $a_n = a_{n+1}\ \forall\ n \geq 3$ & $a_3 = \frac{a_1 + a_2}{2} \Rightarrow a_2 = 179$

4) $\ln x = a x$ has exactly two solutions

$\frac{\ln x}{x} = a$ has exactly two solutions

Let $f(x) = \frac{\ln x}{x}$

Range of $y \in \left(-\infty, \frac{1}{e}\right)$ $\therefore\ a \in \left(0, \frac{1}{e}\right)$

5) Conceptual

6) Let image of A about $y = x, y = -2x$ be P & Q. Then P & Q will lie on side BC.

$\therefore\ P = (2,1), Q = \left(\frac{-11}{5}, \frac{2}{5}\right)$

$\therefore$ Equation of $BC : x - 7y + 5 = 0$

$\therefore\ r = \perp^r$ distance from $I(0,0)$ to $BC = \frac{1}{\sqrt{2}}$

7) $(x \cot y\, dy + \ln(\sin y)\, dx) + (\ln(\cos x)\, dy - y \tan x\, dx) = 0$

$d(x\ln(\sin y)) + d(y\ln(\cos x)) = 0$

$x\ln(\sin y) + y\ln(\cos x) = \ln(k)$

$(\sin y)^x (\cos x)^y = c$

8) ${}^{\left({}^{n}C_2\right)}C_2 - n.{}^{n-1}C_2$

9) $a^2 + b^2 + c^2 + d^2 - ab - bc - cd - d + \frac{2}{5}$

$$= \left(a-\frac{b}{2}\right)^2 + \frac{3}{4}\left(b-\frac{2c}{3}\right)^2 + \frac{2}{3}\left(c-\frac{3d}{4}\right)^2 + \frac{5}{8}\left(d-\frac{4}{5}\right)^2$$

$$\Rightarrow d=\frac{4}{5}, c=\frac{3}{5}, b=\frac{2}{5}, a=\frac{1}{5}$$

10) $x^2-x+1=\left(x-\frac{1}{2}\right)^2+\frac{3}{4}>0\ \forall\ x\in R$

$(x^2-x+1)^{x-1}<1 \Rightarrow (x-1)\log(x^2-x+1)<0$

Casa 1: $(x-1)>0 \Rightarrow \log(x^2-x+1)<0$

$x\in(0,1) \Rightarrow$ no solution

Case 2: $x-1<0 \Rightarrow \log(x^2-x+1)>0$

$x\in(-\infty,1)\cap x\in(-\infty,0)\cup(1,\infty)$

$x\in(-\infty,0)$

11) $g'(x)=f'(\tan x)\sec^2 x-f'(\cot x)\operatorname{cosec}^2 x$

$\because f''(x)<0 \Rightarrow f'(x)$ is decreasing

$\tan x<\cot x\ \forall\ x\in\left(\frac{\pi}{2},\frac{3\pi}{4}\right)$

$\therefore f'(\tan x)>f'(\cot x)\ \&\ \sec^2 x>\operatorname{cosec}^2 x\ \forall\ x\in\left(\frac{\pi}{2},\frac{3\pi}{4}\right)$

$\Rightarrow g(x)$ is increasing in $\left(\frac{\pi}{2},\frac{3\pi}{4}\right)$

$g(x)$ is decreasing in $\left(\frac{3\pi}{4},\pi\right)$ and $g(x)$ has local maximum at $x=\frac{3\pi}{4}$

12) $\frac{dy}{dx}=3t$

Equation of tangent at 't' is $(y-4t^3)=3t(x-2t^2-1)$ ....(1)

This is a normal at another point 't_1'

$\therefore$ Slope of normal at $(2t_1^2+1, 4t_1^3)=3t$

i.e., $\frac{-1}{3t_1}=3t$ ....(2)

$\&\ (2t_1^2+1, 4t_1^3)$ lies on (1)

$\Rightarrow t_1 = \frac{1}{2}$ ....(3)

From (2) & (3), $t = \pm \frac{\sqrt{2}}{3}$

$\therefore$ Required straight lines are $\sqrt{2}x - y = \frac{31\sqrt{2}}{27}$ & $\sqrt{2}x + y = \frac{31\sqrt{2}}{27}$

13) $f(x+1) = \frac{f(x)-5}{f(x)-3}$ ....(1)

$\Rightarrow f(x) = \frac{3f(x+1)-5}{f(x+1)-1}$

Replace 'x' by $x \ 1$

$\Rightarrow f(x-1) = \frac{3f(x)-5}{f(x)-1}$ ....(2)

Replace 'x' by $x+1$ in (1)

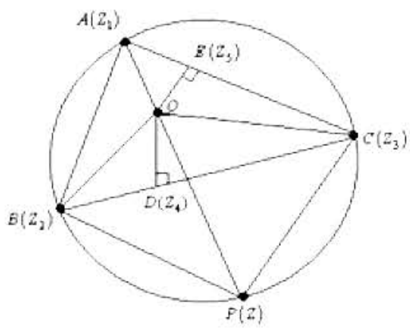
$f(x+2) = \frac{f(x+1)-5}{f(x+1)-3} = \frac{2f(x)-5}{f(x)-2}$ ....(3)

Similarly, $f(x-2) = \frac{2f(x)-5}{f(x)-2}$

$\therefore f(x+2) = f(x-2) \Rightarrow f(x+4) = f(x)$

P)

14)



$CE = CD, OD = OE$

In $\Delta ODC, \frac{0-Z_4}{Z_3-Z_4} = \frac{OD}{CD} e^{i\pi/2}$ ....(1)

In $\Delta OCE, \frac{Z_3 - Z_5}{0 - Z_5} = \frac{OC}{OE} e^{iA/2}$ ....(2)

$(1)\times(2) \Rightarrow Z_3 = \frac{2}{\frac{1}{Z_4} + \frac{1}{Z_5}}$

15) $\angle CBP = \angle CAP = \frac{A}{2}$

$\angle CBO = \frac{B}{2}$

$\therefore \angle OBP = \frac{A+B}{2}$ ....(1)

$\angle BAO = \frac{A}{2}, \angle ABO = \frac{B}{2}$

$\therefore \angle BOP = \frac{A+B}{2}$ ....(2)

$\therefore$ From (1) & (2) $BP = OP$

$\therefore |Z - Z_2| = |Z - 0|$

Similarly, $|Z - Z_3| = |Z - 0|$

$\therefore \frac{|Z - Z_2|}{|Z|} + \frac{|Z|}{|Z - Z_3|} = 2$

16) In $\Delta BOP, \frac{Z_2 - Z}{0 - Z} = \frac{BP}{OP} e^{ic}$

In $\Delta CED, \frac{Z_5 - Z_3}{Z_4 - Z_3} - \frac{CD}{CE} e^{-ic}$

$\therefore \left(\frac{Z - Z_2}{Z}\right)\left(\frac{Z_3 - Z_5}{Z_3 - Z_4}\right) = 1$

P)

17) Area of $\Delta DHP - \frac{1}{2} DH^2 \sin 2C$

$= \frac{1}{2}(2R\cos B)^2 \sin 2C$

$= 2R^2 \cos^2 B \sin 2C$

18) In $\Delta AHC, \sin(90^0 - A) = \frac{AH}{2R_1}$ (R_1 is circum radius of ΔAHC)

$\cos A = \frac{2R\cos A}{2R_1}$

$\therefore R_1 = R$

Ratio $= \dfrac{2\sin\frac{\pi}{n}}{2\tan\frac{\pi}{n}} = \cos\frac{\pi}{n}$

Which is least when $n = 3\ (as\ n \ge 3) \Rightarrow a = \frac{1}{2}$

$\left(\frac{1}{2}, 2\right)$ lies inside the circle with centre $\left(\frac{1}{2}, 1\right)$ & radius $= 3$

$2(r_1 + r_2) = 2R \Rightarrow r_1 + r_2 = R = 3$

20) $n_1 = L.C.M.(2,3,4,\dots 9) - 1$

$n_2 = 2 \times L.C.M.(2,3,4,\dots 9) - 1$

$n_{2008} = 2008 \times L.C.M.(2,3,4,\dots 9) - 1 = 2008 \times 2520 - 1$

$\therefore$ Remainder when n_{2008} is divided by 11 is 5.

21) The number of planes which have three points on one side & the fourth point on the other side $= 4$.

The number of planes which have two points on each side of the plane $= 3$

22) $\gamma\gamma\gamma = \gamma(111)$

$= \gamma.3.37 = (\alpha\beta)(\gamma\beta)$

Clearly, γ is equal to the last digit of β^2. Now 37 must divide one of the $\alpha\beta$ or $\gamma\beta$

Let it be $\alpha\beta$ then $\alpha\beta = 37\ or\ 74\ (if\ \gamma = 8,\ then\ \gamma.3.37 = 12.74)$

$\alpha\beta = 74$ is not possible because

$\left.\begin{array}{l}(\alpha\beta)(\gamma\beta) \ge 74.14 = 1036 \\ But\ \gamma(111) = 12.74\end{array}\right\}\ for\ \gamma = 8$

$\therefore\ \alpha\beta = 37 \Rightarrow \delta\beta = 27 \Rightarrow \beta = 7, \alpha = 3, \delta = 2\ \&\ \gamma = 9$

$\therefore\ \alpha + \beta + \gamma + \delta = 21$

23) Let $[x] \ge 2$

Given $\dfrac{[x]}{x - [x]} = \dfrac{x}{[x]}$

Always $\dfrac{x}{[x]} < 2,$

If $[x] \ge 2, \dfrac{[x]}{x - [x]} > 2$

$\therefore [x]=1$

$$\frac{1}{x-1}=\frac{x}{1} \Rightarrow x^2-x-1=0$$

$$x=\frac{1+\sqrt{5}}{2}$$

$\therefore 2x=1+\sqrt{5}$

$[x]=1$

$2x-2[x]+1=1+\sqrt{5}-2+1=\sqrt{5}$

24) In Δ^{les} AIF, AIE

$$\frac{IF}{\sin\frac{A}{2}}=AI=\frac{IE}{\sin\frac{A}{2}} \Rightarrow AI^2=\frac{IE.IF}{\sin^2\frac{A}{2}} \Rightarrow \frac{IA.IB.IC}{ID.IE.IF}=\frac{1}{\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}}=\frac{4R}{r}=10$$

25) Area $=16-4\int_0^{\ln 2}\left(e^{-x}-\frac{1}{2}\right)$

$\because |x|\le 2,\ |y|\le 2,\ |x|+|y|\le 4$

26) $(abc+bcd+cda+dab)^{10}=(abcd)^{10}\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}\right)^{10}$

Coefficient of $a^8b^4c^9d^9$ is $\frac{10!}{2!6!1!1!}$

27) $99\times 97\times 95\times\ldots\ldots\times 51=\frac{100!}{100\times 98\times\ldots\ldots\times 52}\times\frac{1}{50!}=\frac{100!\,25!}{2^{25}\times 50!\times 50!}$

Exponent of 3 in $100!=48$

Exponent of 3 in $50!=22$

Exponent of 3 in $25!=10$

$\therefore P=48+10-(2\times 22)=14=2K \Rightarrow K=7$

28) Any point on the line $x+y=1$ can be taken $(t,1-t)$

Equation of the chord, with this as mid point is $y(1-t)-2a(x+t)=(1-t)^2-4at$.

It passes through $(a,2a)$.

So, $t^2-2t+2a^2-2a+1=0$, this should have 2 distinct real roots

so $a^2-a<0,\ 0<a<1$, so length of latusrectum <4