

Class XII (Mathematics)

Time: 10:15 A.M.- 1:30 P.M.

Mock Board Exam (I) -2016

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General Instructions:

- (i) Please check that this question paper contains **04** printed pages.
- (ii) All questions are compulsory.
- (iii) The question paper consists of **26** questions.
- (iv) **Please write down the Serial Number of the question before attempting it.**
- (v) 15-minute time has been allotted to read this question paper, during this period students will read the question paper only and will not write any answer on the answer-book.
- (vi) Questions **1-6** in Section A are very short-answer type questions carrying **1** mark each.
- (vii) Questions **7-19** in Section B are long-answer **I** type questions carrying **4** marks each.
- (viii) Questions **20-26** in Section C are long-answer **II** type questions carrying **6** marks each.

Section – A

(Questions number 1 to 6 carry 1 marks each)

1. If the matrix $\begin{bmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix}$ is a skew symmetric matrix then find the value of a , b and c .

2. Write the value(s) of x , if $\begin{bmatrix} 1 & x & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = 0$.

3. If $|A| = \begin{vmatrix} -a(b^2 + c^2 - a^2) & 2b^3 & 2c^3 \\ 2a^3 & -b(c^2 + a^2 - b^2) & 2c^3 \\ 2a^3 & 2b^3 & -c(a^2 + b^2 - c^2) \end{vmatrix} = abc(a^2 + b^2 + c^2)^3$, then find $|\text{adj } A|$.

4. If $f(x) = (25 - x^4)^{1/4}$ for $0 < x < \sqrt{5}$, then find $f\left(f\left(\frac{1}{2}\right)\right)$.

5. The principal value of $\cos^{-1} \left\{ \frac{1}{\sqrt{2}} \left(\cos \frac{9\pi}{10} - \sin \frac{9\pi}{10} \right) \right\}$.
6. Vectors \vec{a} and \vec{b} are such that $|\vec{a}| = \sqrt{3}$, $|\vec{b}| = \frac{2}{3}$ and $(\vec{a} \times \vec{b})$ is unit vector.
Write the angle between \vec{a} and \vec{b} .

Section-B

(Questions number 7 to 19 carry 4 marks each.)

7. Let $f : R \rightarrow R$ and $f(x) = \begin{cases} |x| & ; x \neq 0 \\ x & ; x = 0 \\ 0 & ; \end{cases}$ and $g(x) = [x]$, then does $f \circ g$ and $g \circ f$ coincide in $[-1, 0)$?

8. Find the value of $\tan \left(\frac{1}{2} \cos^{-1} \frac{\sqrt{5}}{3} \right)$. **OR**

Prove that $2 \tan^{-1} \left(\frac{1}{5} \right) + \sec^{-1} \left(\frac{5\sqrt{2}}{7} \right) + 2 \tan^{-1} \left(\frac{1}{8} \right) = \frac{\pi}{4}$.

9. Prove by using properties of determinants:

$$\begin{vmatrix} -a(b^2 + c^2 - a^2) & 2b^3 & 2c^3 \\ 2a^3 & -b(c^2 + a^2 - b^2) & 2c^3 \\ 2a^3 & 2b^3 & -c(a^2 + b^2 - c^2) \end{vmatrix} = abc(a^2 + b^2 + c^2)^3.$$

10. Discuss the continuity of function when $f(x) = \begin{cases} \frac{\sin [x]}{[x]} & ; \text{when } x \neq 0 \\ 0 & ; \text{when } x = 0 \end{cases}$. Where $[.]$ denotes the greatest integer function.

11. If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$. Then prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$.

12. A bag contains 4 balls. Two balls are drawn at random, and are found to be blue. What is the probability that all the balls are blue?

13. Solve : $\frac{dy}{dx} - \frac{1}{x} y = 2x^2$.

14. Evaluate $\int \frac{x^2 - 1}{x\sqrt{x^4 + 3x^2 + 1}} dx$

15. Find the interval in which the function $f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11$ is
- increasing
 - decreasing.

OR

Prove that $f(x) = \frac{4 \sin x}{2 + \cos x}$ is an increasing function of x in $\left[0, \frac{\pi}{2}\right]$.

16. Evaluate $\int \frac{\cos 5x + \cos 4x}{1 - 2 \cos 3x} dx$.

17. Evaluate $\int_{-1}^{1.5} |x \sin(x)| dx$

18. Find a particular solution of the differential equation $(x+1) \frac{dy}{dx} = 2e^{-y} - 1$, given that $y = 0$ when $x = 0$.

OR

Solve the differential equation $\left(1 + e^{\frac{x}{y}}\right) dx + e^{\frac{x}{y}} \left(1 - \frac{x}{y}\right) dy = 0$.

19. $\vec{a}, \vec{b}, \vec{c}$ are the unit vectors. Suppose $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c} = 0$ and angle between \vec{b} and \vec{c} is $\frac{\pi}{6}$, prove that $\vec{a} = \pm 2(\vec{b} \times \vec{c})$.

OR

If $\vec{a} = 5\hat{i} - \hat{j} - 3\hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} - 5\hat{k}$, then show that the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are orthogonal.

Section-C

(Question number 20 to 26 carry 6 marks each)

20. Using integration find the area of the region included between the curves $y = x^2 + 1, y = x, x = 0$ and $y = 2$.

OR

Find the area of the region bounded by the line $y = 3x + 2$, the x-axis and the ordinates $x = -2$ to $x = 3$.

21. If $A = \begin{bmatrix} \cos n & \sin n \\ -\sin n & \cos n \end{bmatrix}$, then prove that $A^n = \begin{bmatrix} \cos n_n & \sin n_n \\ -\sin n_n & \cos n_n \end{bmatrix}$.

22. If length of three sides of a trapezium other than base are equal to 10cm, then find the area of the trapezium when it is maximum.
23. Find the coordinate of the foot of the perpendicular and the length of the perpendicular drawn from the point $P(5, 4, 2)$ to the line $\vec{r} = -\hat{i} + 3\hat{j} + \hat{k} + \lambda(2\hat{i} + 3\hat{j} - \hat{k})$. Also find the image of P in this line.
24. There are two factories located one at place P and the other at place Q. From these locations, a certain commodity is to be delivered to each of the three depots situated at A, B and C. The weekly requirements of the depots are respectively 5, 5 and 4 units of the commodity while the production capacity of the factories at P and Q are respectively 8 and 6 units. The cost of transportation per unit is given below:

From/To	Cost (in Rs)		
	A	B	C
P	160	100	150
Q	100	120	100

How many units should be transported from each factory to each depot in order that the transportation cost is minimum. What will be the minimum transportation cost?

25. From a lot of 10 bulbs, which include 3 defectives, a sample of 2 bulbs is drawn at random. Find the probability distribution of the number of defective bulbs.

OR

In a game, a man wins Rs.1Lakh for a one and loses an Rs.50, 000 for any other number when a fair die is thrown. The man decided to throw a die thrice but to quit as and when he gets a one. Find the expected value of the amount he win/loses.

26. Find the equation of the plane passing through the point $P(1, 1, 1)$ and containing the line $\vec{r} = -3\hat{i} + 1\hat{j} + 5\hat{k} + \lambda(3\hat{i} - 1\hat{j} - 5\hat{k})$. Also show that the plane contains the line $\vec{r} = -\hat{i} + 2\hat{j} + 5\hat{k} + \mu(\hat{i} - 2\hat{j} - 5\hat{k})$.

All the best for next test series